

B.E. Ist Semester Examination,  
 May – 2009  
 Mathematics – 1  
 Paper – Math – 1

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Note: Attempt five questions in all, selecting two questions from each part.

1. (a) Test the convergence or divergence of the series : 6

$$\sum | \sqrt{n^4 + 1} - \sqrt{n^4 - 1} |$$

- (b) Discuss the convergence of the series 7

$$x + \frac{2^2 x^2}{2} + \frac{3^3 x^3}{3} + \frac{4^4 x^4}{4} + \dots \infty (x > 0)$$

- (c) State, with reasons, the values of x for which the series.

$$X - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty \text{ converges}$$

2. (a) Compute to four decimal places, the value of  $\cos 32^\circ$ , by use to Taylor's series. 6

- (b) If p the radius of curvature at any point P on the parabola  $y^2 = 4ax$  and S be its focus, then show that  $p^2$  varies as  $(SP)^3$ . 8

- (c) Show that the asymptotes of the curve  $x^2 y^2 = a^2 (x^2 + y^2)$  form a square of side 2a. 6

3. (a) If  $u = x^2 \tan^{-1} (y/x) - y^2 \tan^{-1} (x/y)$ , evaluate 10

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

- (b) If  $f(x, y) = \tan^{-1} (xy)$ , compute  $f(0.9, -1.2)$  approximately. 10

4. Find the maximum and minimum distances from the origin to the curve  $5x^2 + 6xy + 5y^2 - 8 = 0$ .

- (b) Evaluate  $\int_0^{\alpha} \frac{\log(1 + \alpha x)}{1 + x^2} dx$  10

**Part – B**

5. (a) Find the volume of solid formed by revolving a loop of the lemniscate  $r^2 = a^2 \cos 2\theta$  about the initial line 10

- (b) Evaluate  $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$  by changing the order of integration. 10

6. (a) Evaluate  $\iiint (x + y + z) \, dx \, dy \, dz$  over the tetrahedron bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and  $x + y + z = 1$ . 10

(b) Show that  $\int_0^{\pi/2} \sqrt{\sin \theta} \, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$ . 10

7.(a) Find the constants  $a$  and  $b$  so that the surface  $ax^2 - byz = (a + 2)x$  is orthogonal to the surface  $4x^2y + z^3 = 4$  at the point  $(1, -1, 2)$  10

(b) If  $v_1$  and  $v_2$  be the vectors joining the fixed points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  respectively to a variable point  $(x, y, z)$ , prove that 10

8. (a) verify stoke's theorem for  $F = (x^2 + y^2) \mathbf{i} - 2xy \mathbf{j}$  taken around the rectangle bounded by the lines  $x = \pm a$ ,  $y = 0$ ,  $y = b$  10

(b) Using divergence theorem, evaluate  $\int_S \mathbf{R} \cdot \mathbf{N} \, ds$  where  $S$  is the surface of the sphere  $x^2 + y^2 + z^2 = 9$  10